Optimal Electricity Procurement Enabled by Privacy-Preserving Samples

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Ubiquitous Samples in Power System Operation

- The legacy grid has been working admirably over a century.
- New components are coming and reshaping everything.
- Data/Samples in power system is almost everywhere.
- The profusion of data paves the way for smart grid operations and electricity market optimization.

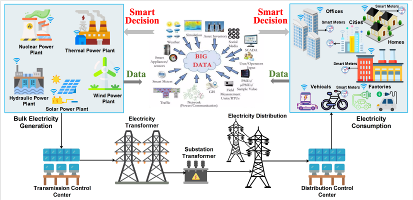


Figure: Samples are Changing Traditional Power System Operation.

Question: How Do Samples Facilitate Decision-Making Involving Distribution Estimations?

The Impact of Samples on Decision-making

- Optimal solutions rely on key distribution knowledge.
- More samples improve distribution characterization.

- How many samples are sufficient to ensure “good” solutions?
- If there exist noises in samples, e.g., measurement noises or DP-based noise injection, what is the impact on the minimum required number of samples?

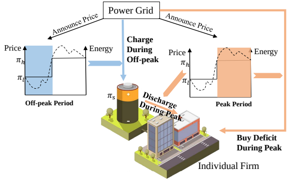


Figure: Electricity Storage (electricity demand distribution).

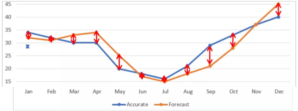


Figure: Electricity Procurement (forecast error distribution).

Electricity Procurement Model

The mathematical formulation:

$$Q_i = \pi_d \cdot S_i + \pi_{eq} \cdot M_i \tag{1}$$

$$\pi_{eq} = \begin{cases} \pi_u, & M > 0 \\ 0, & M = 0 \\ \pi_o, & M < 0 \end{cases} \tag{2}$$

- π_d: the electricity price in the day-ahead market;
- $\hat{L}_i$: the forecast electricity demand for player i;
- s_i: the strategic electricity procurement for player i;
- e_i: the demand forecast error, $e_i = \hat{L}_i - L_i$, where L_i is the actual demand;
- π_{eq}: the trading electricity price in the real-time market.

$$S_i = \hat{L}_i - s_i,\ M_i = s_i - e_i,\ Q = \sum_{i=1}^{N} Q_i \tag{3}$$

Electricity Procurement Model

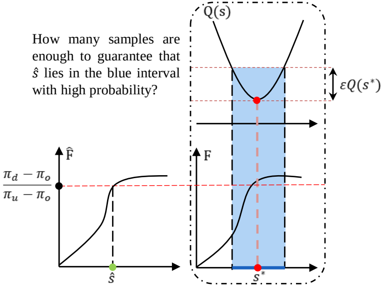
Lemma (The Optimal Strategy)

If $\frac{\partial \mathbb{E}\{e_i|e=\beta\}}{\partial \beta} \geq 0, \forall i$ *holds, then the optimal strategy is unique and given by*

$$s_i^* = \mathbb{E}\{e_i|e = s^*\}, \tag{4}$$

where

$$s^* = F_e^{-1}\left(\frac{\pi_d - \pi_o}{\pi_u - \pi_o}\right). \tag{5}$$



- **Authentic Samples:** $\{x_1, x_2, \cdots, x_N\}$;
- **Privacy-Preserving Samples** $\{x_1 + r_1, x_2 + r_2, \cdots, x_N + r_N\}$;

Theoretical Analysis

Using Bernsterin's inequality and some mathematical tricks, we can prove that (under some conditions and corrections)

Theorem (Authentic Case)

For any $\varepsilon > 0$, with probability at least δ, the SAA-based solution $\hat{s}^{\phi}$ satisfies $Q(\hat{s}^{\phi}) - Q(s^*) \leq \varepsilon |Q(s^*)|$ given N samples, where

$$N \geq \frac{4 \log\left(\frac{2}{1-\delta}\right)}{\varepsilon |Q(s^*)| f(s^*) \eta}. \tag{6}$$

Theorem (Privacy-preserving Case)

For any $\varepsilon > 0$, with probability at least δ, the SAA-based solution $\widetilde{s}^{\phi}$ satisfies $Q(\tilde{s}^{\phi}) - Q(s^*) \leq \varepsilon |Q(s^*)|$ given N samples, where

$$N \geq \frac{12 + 4\kappa(\pi_u - \pi_o)/(\pi_u - \pi_d)}{3\varepsilon |Q(s^*)| f(s^*) \eta} \log\left(\frac{2}{1-\delta}\right). \tag{7}$$

Performance Analysis

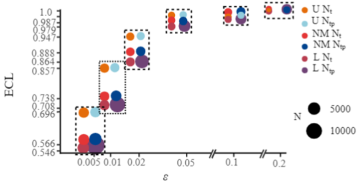


Fig. 4. Sample complexity comparison (Lemma 3.3 & Theorem 4.6) for varying performance guarantees. N_t and N_{tp} illustrate the number of samples obtained from Lemma 3.3 and Theorem 4.6, respectively. In each dashed box, ε is the same.

TABLE II
SAMPLE COMPLEXITY ANALYSIS FOR DIFFERENT PRIVACY LEVELS.

ϵ	N_{bp}	N_{tp}	ϵ	N_{bp}	N_{tp}
0.001	2.39E+08	1.57E+06	0.1	9.33E+04	594
0.01	2.46E+06	1.62E+04	0.2	7.53E+04	476
0.05	1.65E+05	1.07E+03	0.5	7.03E+04	443

- When $\epsilon \geq 0.1$, the change rate of the required theoretical number of samples is slower.

Theoretical bounds provide quantitative analysis for required samples to ensure the performance of decision-making.

Ref:
1. Jiang, W., Huang, J., Xu, G., & Wu, C. (2023). Sample-Oriented Electricity Storage Sharing Mechanism Design With Performance Guarantees. IEEE Transactions on Smart Grid.
2. Jiang, W. & Wu, C. (2023). Optimal Electricity Procurement Enabled by Privacy-Preserving Samples. Revision submitted to IEEE Transactions on Energy Markets, Policy, and Regulation.

Thank you for your listening!

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