Consumer-based Carbon Costs: Integrating Consumer Carbon Preferences in Electricity Markets

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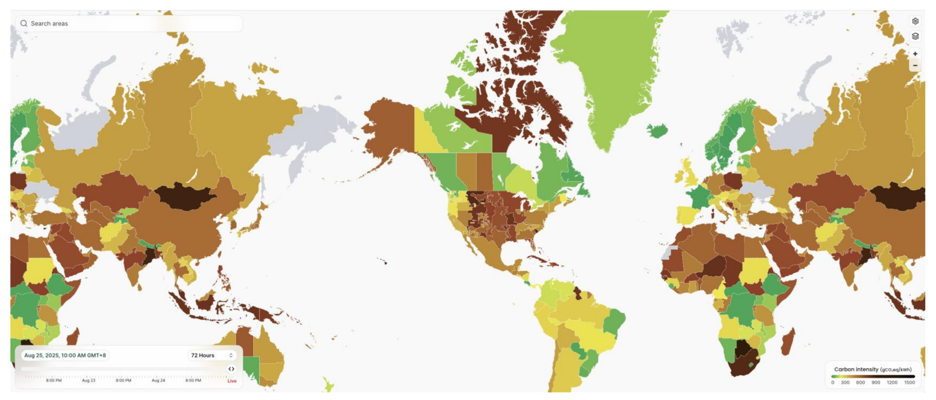
Introduction



source: Electricity Maps (accessed on Aug. 24, 2025). https://app.electricitymaps.com/map/72h/hourly

Carbon emissions vary with time and/or location.

Green hydrogen now costs USD 4-6/kilogram (kg), 2-3 times more than grey hydrogen. The largest

IRENA (2021), ***Making the breakthrough: Green hydrogen policies and technology costs***, International Renewable Energy Agency, Abu Dhabi.

National Grid "Green Light Signal" source: https://www.nationalgrid.com/greenlightsignal

An increasing number of carbon-sensitive loads.

- Voluntarily
- To receive subsidies or higher prices

Electricity Market Clearing with Carbon Costs

Core Idea: Internalize the consideration of consumer carbon preferences into the electricity market!

- Consumer-defined carbon costs
  – Reflect how much revenue the consumer is willing to "forgo" to avoid carbon emissions.
- Carbon allocation mechanism
  – Allocate carbon emissions directly from generators to consumers without considering the physical power grid.

Carbon-aware Objective

$$\max_{P_G,P_D,\theta,\pi,E_D} \; u_D^\intercal P_D - c_D^\intercal E_D - c_G^\intercal P_G \tag{1a}$$

Standard DCOPF Constraints

$$\text{s.t.} \sum_{d\in\mathcal{D}_i} P_{D,d} + \sum_{j:(i,j)\in\mathcal{L}} \beta_{ij}(\theta_i-\theta_j) = \sum_{g\in\mathcal{G}_i} P_{G,g}, \quad \forall i\in\mathcal{N}, \;:\; \lambda_{P,i} \tag{1b}$$

$$\beta_{ij}(\theta_i-\theta_j) \le F_{ij}^{\text{lim}}, \quad \forall (i,j)\in\mathcal{L}, \;:\; \overline{\eta}_{L,ij} \tag{1c}$$

$$\beta_{ij}(\theta_i-\theta_j) \ge -F_{ij}^{\text{lim}}, \quad \forall (i,j)\in\mathcal{L}, \;:\; \underline{\eta}_{L,ij} \tag{1d}$$

$$P_{G,g}^{\text{min}} \le P_{G,g} \le P_{G,g}^{\text{max}}, \quad \forall g\in\mathcal{G}, \;:\; \overline{\eta}_{G,g}, \underline{\eta}_{G,g} \tag{1e}$$

$$P_{D,d}^{\text{min}} \le P_{D,d} \le P_{D,d}^{\text{max}}, \quad \forall d\in\mathcal{D}, \;:\; \overline{\eta}_{D,d}, \underline{\eta}_{D,d} \tag{1f}$$

$$\theta_{\text{ref}} = 0, \tag{1g}$$

Carbon Allocation

$$\sum_{d\in\mathcal{D}} \pi_{g,d} = P_{G,g}, \quad \forall g\in\mathcal{G}, \;:\; \lambda_{G,g} \tag{1h}$$

$$\sum_{g\in\mathcal{G}} \pi_{g,d} = P_{D,d}, \quad \forall d\in\mathcal{D}, \;:\; \lambda_{D,d} \tag{1i}$$

$$\sum_{g\in\mathcal{G}} e_{G,g}\pi_{g,d} = E_{D,d}, \quad \forall d\in\mathcal{D}, \;:\; \lambda_{E,d} \tag{1j}$$

$$\pi_{g,d} \ge 0, \quad \forall g\in\mathcal{G}, \; \forall d\in\mathcal{D}. \tag{1k}$$

Interesting Observations

Observation 1: Carbon allocation mechanism drives the whole system to allocate lower carbon power to consumers with higher carbon costs.

Observation 2: Our model generalizes two standard market clearing models as special cases.

- Equivalent to standard (carbon-agnostic) market clearing when all carbon costs are zero.
- Equivalent to carbon tax on generation when all carbon costs are identical.

Equivalent Equilibrium Formulation

Generators: Profit maximization problem

$$\max_{P_{G,g}} \; (\lambda_{P,i:g\in\mathcal{G}_i} + \lambda_{G,g} - c_{G,g}) \cdot P_{G,g}$$
$$\text{s.t. } P_{G,g}^{\text{min}} \le P_{G,g} \le P_{G,g}^{\text{max}}.$$

Consumers: Utility maximization problem

$$\max_{P_{D,d}} \; (u_{D,d} - \lambda_{P,i:d\in\mathcal{D}_i} + \lambda_{D,d}) \cdot P_{D,d}$$
$$\text{s.t. } P_{D,d}^{\text{min}} \le P_{D,d} \le P_{D,d}^{\text{max}}.$$

Transmission Owner: Profit maximization problem

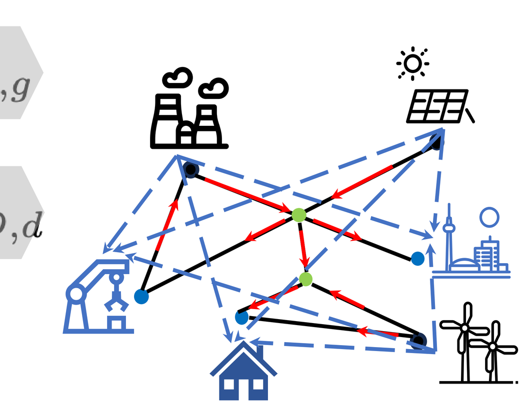
$$\max_{\theta} \; \sum_i \lambda_{P,i} \cdot \left(\sum_{j,(i,j)\in\mathcal{L}} \beta_{ij}(\theta_j-\theta_i) \right)$$
$$\text{s.t. } -F_{ij}^{\text{lim}} \le \beta_{ij}(\theta_i-\theta_j) \le F_{ij}^{\text{lim}}, \quad \forall (i,j)\in\mathcal{L},$$
$$\theta_{ref} = 0,$$

Price Setter: Enforce the nodal power balance constraint

$$\sum_{d\in\mathcal{D}_i} P_{D,d} + \sum_{j:(i,j)\in\mathcal{L}} \beta_{ij}(\theta_i-\theta_j) = \sum_{g\in\mathcal{G}_i} P_{G,g}, \quad : \lambda_{P,i},$$

Carbon Manager: Total carbon cost minimization problem

$$\max_{\pi,E_D} \; -c_D^\intercal E_D$$
$$\text{s.t. } \sum_{d\in\mathcal{D}} \pi_{g,d} = P_{G,g}, \quad \forall g\in\mathcal{G}, \;:\; \lambda_{G,g}$$
$$\sum_{g\in\mathcal{G}} \pi_{g,d} = P_{D,d}, \quad \forall d\in\mathcal{D}, \;:\; \lambda_{D,d}$$
$$\sum_{g\in\mathcal{G}} e_{G,g}\pi_{g,d} = E_{D,d}, \forall d\in\mathcal{D},$$
$$\pi_{g,d} \ge 0, \quad \forall g\in\mathcal{G}, \; \forall d\in\mathcal{D}.$$



Carbon Cost Model

Carbon-Adjusted Prices

Definition (Carbon-Adjusted Prices): The carbon-adjusted prices are defined as

$$\lambda_{P,i:g\in\mathcal{G}_i} + \lambda_{G,g} \quad \textit{for generators } g\in\mathcal{G},$$
$$\lambda_{P,i:d\in\mathcal{D}_i} - \lambda_{D,d} \quad \textit{for consumers } d\in\mathcal{D}.$$

Theorem (Ordering of Carbon-Adjustments): *For a set of generators $\mathcal{G}$ with increasing emission factors $e_{G,(1)} \le e_{G,(2)} \le \cdots \le e_{G,(|\mathcal{G}|)}$, the corresponding generator carbon-adjustments will be decreasing,*

$$\lambda_{G,(1)} \ge \lambda_{G,(2)} \ge \cdots \ge \lambda_{G,(|\mathcal{G}|)}.$$

For a set of consumers $\mathcal{D}$ with decreasing carbon-costs $c_{D,(1)} \ge c_{D,(2)} \ge \cdots \ge c_{D,(|\mathcal{D}|)}$, the corresponding consumer carbon-adjustments will be increasing,

$$\lambda_{D,(1)} \le \lambda_{D,(2)} \le \cdots \le \lambda_{D,(|\mathcal{D}|)}$$

Remark: Our carbon cost model is fair in two senses that

- the higher-emitting generators are penalized with lower payments;
- the consumers that submit higher carbon costs contribute more to cover the increases in generation cost that arise from prioritizing low carbon generation.

Market-Clearing Properties

Proposition 1: Our model satisfies revenue adequacy, i.e., the payment the ISO receives from consumers is always higher than or equal to their total payments to generators, the transmission owner, and the carbon manager.

Proposition 2: Our model satisfies individual rationality given $P_G^{\text{min}} = P_D^{min} = 0$, i.e., generators and consumers always recover their operational costs and do not incur a loss.

Conclusions and Takeaways

- We propose a new green electricity market clearing model with consumer-based carbon costs (carbon cost model).
- The equilibrium formulation of the carbon cost model gives rise to carbon-adjusted prices. Low-carbon generators and carbon-sensitive consumers will face high carbon-adjusted prices.
- Our carbon cost model satisfies the same desirable market properties as standard markets based on LMP, i.e., revenue adequacy and individual rationality.