Electricity Markets with Consumer-based Carbon Costs: A New Paradigm for Greening the Grid

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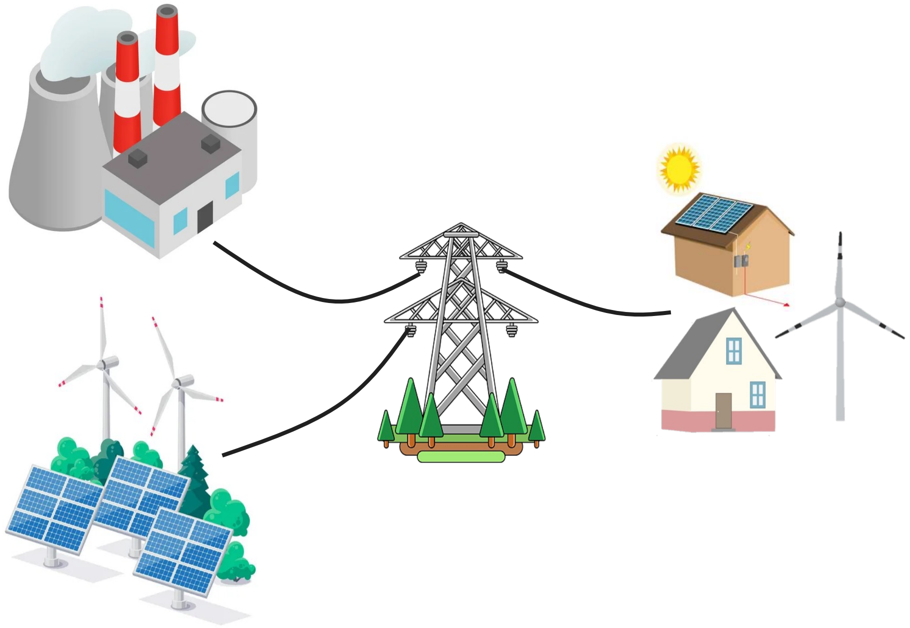
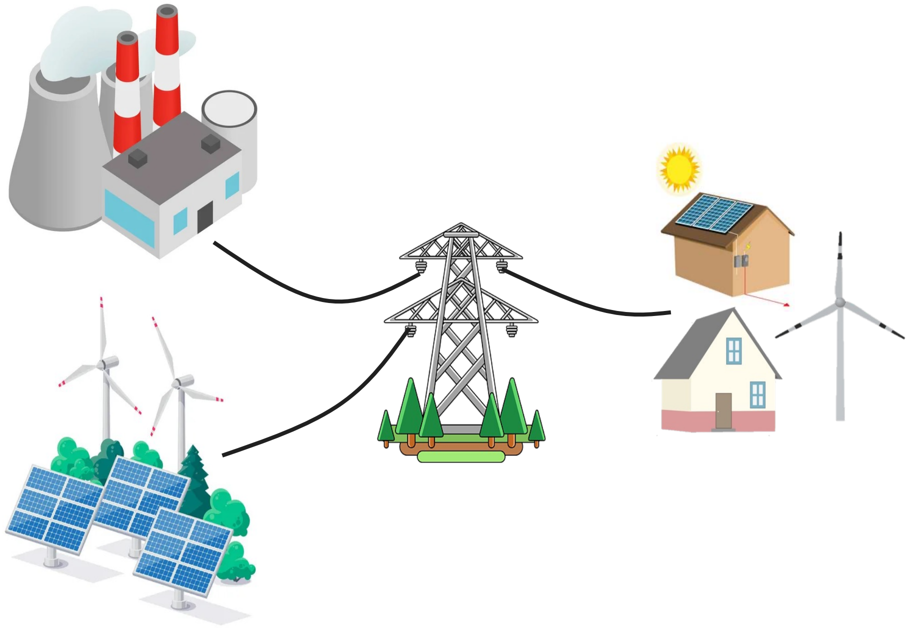
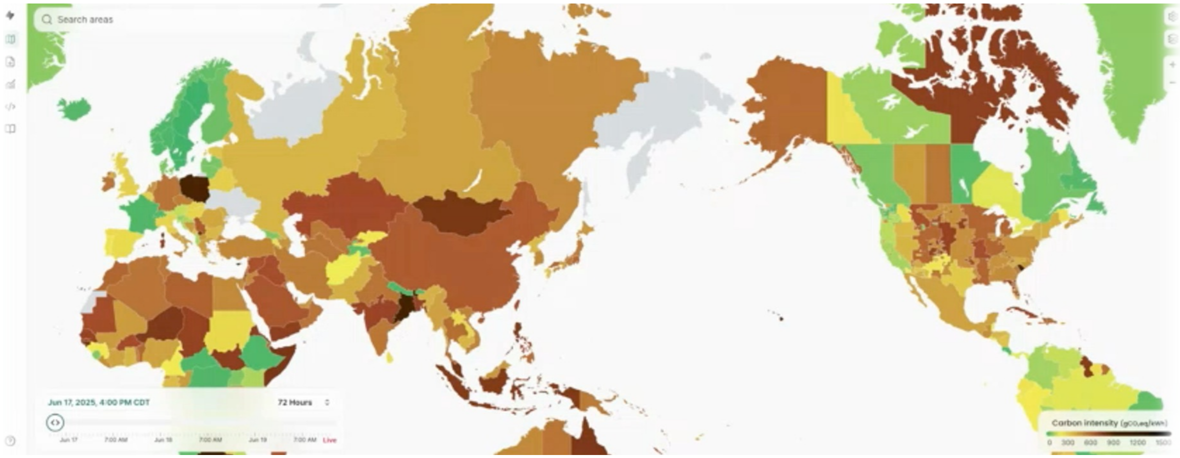
Carbon Emission in Power Systems



Electricity generation produces carbon emissions.

Carbon Emission in Power Systems





source: Electricity Maps (accessed on Oct.25, 2025).
https://app.electricitymaps.com/map/72h/hourly

Carbon emissions **vary with time and/or location**.

Carbon-Sensitive Electrical Loads

- An increasing number of carbon-sensitive loads!

  Electricity consumers are willing to **adapt the timing and/or location of their electricity usage to minimize carbon footprints**.

Carbon-Sensitive Electrical Loads

▸ An increasing number of carbon-sensitive loads!

Electricity consumers are willing to **adapt the timing and/or location of their electricity usage to minimize carbon footprints** (even through paying more).

"Apple uses data that combines grid, emissions, and weather information into one, **easy-to-follow signal**." https://shorturl.at/kvQRV

Google shifts data center loads: https://ieeexplore.ieee.org/document/9770383

National Grid "Green Light Signal" tells consumers **when carbon intensity is low**! https://www.nationalgrid.com/greenlightsignal

Carbon-Sensitive Electrical Loads

▶ An increasing number of carbon-sensitive loads!

Electricity consumers are willing to **adapt the timing and/or location of their electricity usage to minimize carbon footprints** (even through paying more).

- **Voluntarily** (to help mitigate climate change)
- **To receive subsidies** (e.g. clean hydrogen production tax credits) or **higher prices** (green vs grey hydrogen)

Carbon Intensity (kg CO_2e per kg H_2)	Max Hydrogen Production Tax Credit ($/kg H_2)
4–2.5	$0.60
2.5–1.5	$0.75
1.5–0.45	$1.00
<0.45	$3.00

source: https://www.energy.gov/eere/fuelcells/financial-incentives-hydrogen-and-fuel-cell-projects

Green hydrogen now costs USD 4-6/kilogram (kg), 2-3 times more than grey hydrogen. The largest

IRENA (2021), ***Making the breakthrough: Green hydrogen policies and technology costs***, International Renewable Energy Agency, Abu Dhabi.

Carbon-Sensitive Electrical Loads

- An increasing number of carbon-sensitive loads!

  Electricity consumers are willing to **adapt the timing and/or location of their electricity usage to minimize carbon footprints** (even through paying more).

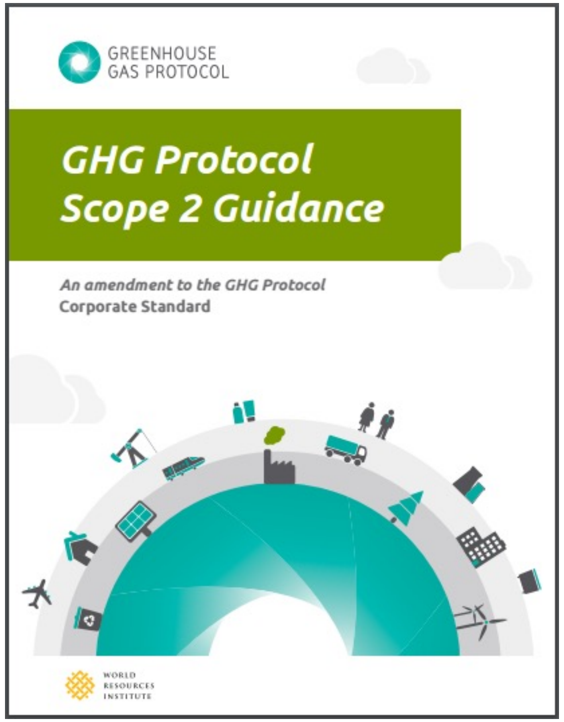
  To enable such carbon-aware practices, carbon-sensitive consumers require that

  - Real-time “carbon signals”
  - An associated framework that supports carbon accounting for electricity usage

Carbon-sensitive consumers

Carbon Emission Accounting of Electricity Usage

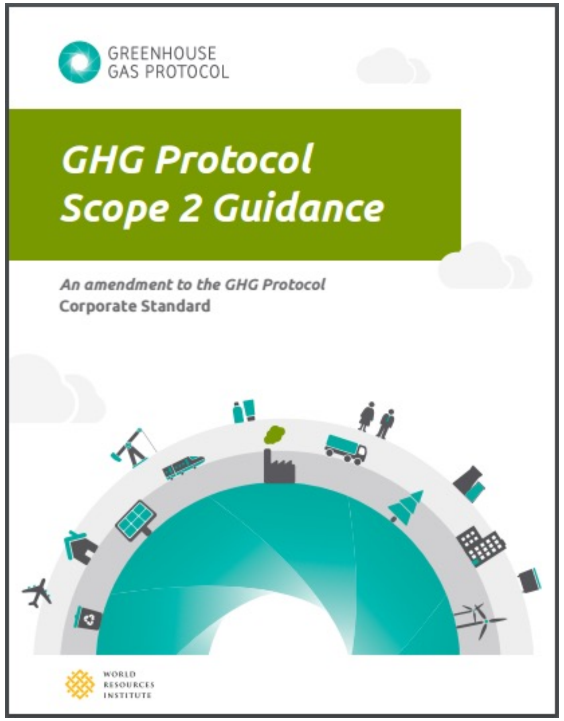
- Market-based methods: Renewable Energy Certificates or Power Purchase Agreements.



Current Carbon Accounting from GHG Protocol

- Market-based methods: Renewable Energy Certificates or Power Purchase Agreements.

- Location-based methods: Average Carbon Emission.



Current Carbon Accounting from GHG Protocol

- Market-based methods: Renewable Energy Certificates

Electricity Market | **Carbon Market**

Separated Electricity Market and Carbon Market (Carbon Accounting)

- Location-based methods: Average Carbon Emission.

Electricity Market → **Carbon Accounting**

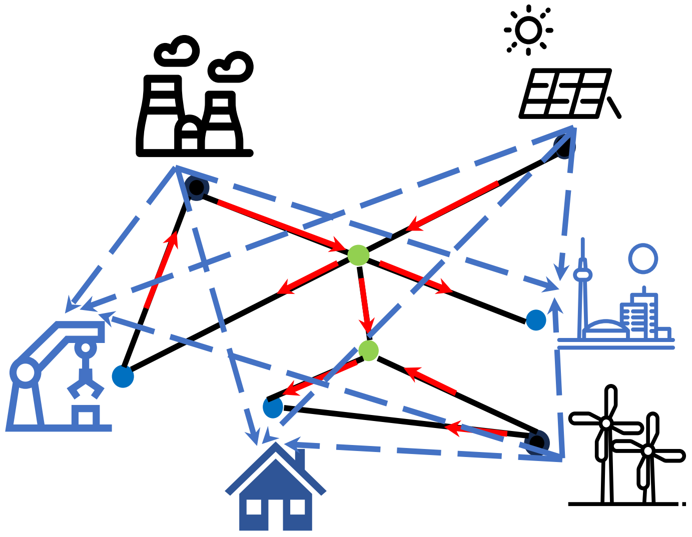
No Carbon Market and Carbon Accounting Happens after Electricity Market Clearing

Can we integrate carbon emissions into the electricity market and simultaneously achieve carbon accounting?

Carbon Cost Model

▶ Core Idea: Introduce consumers' carbon preferences and carbon allocation into the electricity market!

- Consumer-defined carbon costs
  - Reflect how much revenue the consumer is willing to "forgo" to avoid carbon emissions.
- Carbon allocation mechanism
  - Allocate carbon emissions directly from generators to consumers without considering the physical power grid.



Carbon Cost Model

Consumer-based Carbon Costs

Carbon-aware Objective

$$\max_{P_G, P_D, \theta, \pi, E_D} \quad u_D^\intercal P_D - c_D^\intercal E_D - c_G^\intercal P_G \tag{1a}$$

Carbon costs

Standard DCOPF Constraints

$$\text{s.t.} \sum_{d \in \mathcal{D}_i} P_{D,d} + \sum_{j:(i,j) \in \mathcal{L}} \beta_{ij}(\theta_i - \theta_j) = \sum_{g \in \mathcal{G}_i} P_{G,g},$$

$$\forall i \in \mathcal{N}, \tag{1b}$$

Nodal power balance

$$\beta_{ij}(\theta_i - \theta_j) \le F_{ij}^{\text{lim}}, \quad \forall (i,j) \in \mathcal{L}, \tag{1c}$$

$$\beta_{ij}(\theta_i - \theta_j) \ge -F_{ij}^{\text{lim}}, \quad \forall (i,j) \in \mathcal{L}, \tag{1d}$$

Transmission line limit

$$P_{G,g}^{\min} \le P_{G,g} \le P_{G,g}^{\max}, \quad \forall g \in \mathcal{G}, \tag{1e}$$

Generation capacity limit

$$P_{D,d}^{\min} \le P_{D,d} \le P_{D,d}^{\max}, \quad \forall d \in \mathcal{D}, \tag{1f}$$

Demand flexibility limit

$$\theta_{\text{ref}} = 0, \tag{1g}$$

Ref. bus voltage

Carbon Allocation

$$\sum_{d \in \mathcal{D}} \pi_{g,d} = P_{G,g}, \ \forall g \in \mathcal{G}, \tag{1h}$$

$$\sum_{g \in \mathcal{G}} \pi_{g,d} = P_{D,d}, \ \forall d \in \mathcal{D}, \tag{1i}$$

$$\sum_{g \in \mathcal{G}} e_{G,g} \pi_{g,d} = E_{D,d}, \ \forall d \in \mathcal{D}, \tag{1j}$$

$$\pi_{g,d} \ge 0, \ \forall g \in \mathcal{G}, \ \forall d \in \mathcal{D}. \tag{1k}$$

Consumer-based Carbon Costs

Carbon-aware Objective

$$\max_{P_G, P_D, \theta, \pi, E_D} \quad u_D^\intercal P_D - c_D^\intercal E_D - c_G^\intercal P_G \tag{1a}$$

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Carbon Allocation Mechanism

Carbon-aware Objective

$$\max_{P_G, P_D, \theta, \pi, E_D} \quad u_D^\top P_D - c_D^\top E_D - c_G^\top P_G \tag{1a}$$

Maximize social welfare under consumer-defined carbon costs

Standard DCOPF Constraints

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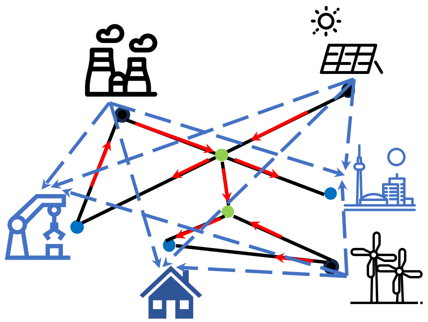

Carbon Cost Model

Carbon Allocation Mechanism

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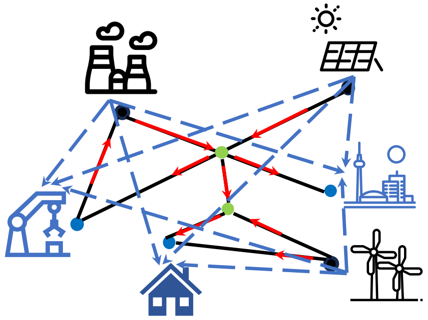

Carbon Cost Model

Carbon Allocation Results

Carbon-aware Objective

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$$\pi_{g,d} \geq 0, \ \forall g \in \mathcal{G}, \ \forall d \in \mathcal{D}. \tag{1k}$$

- Allocate lower carbon power to consumers with higher carbon costs.

Special Versions of Carbon Cost Model

Carbon-aware Objective

$$\max_{P_G, P_D, \theta, \pi, E_D} \quad u_D^\intercal P_D - c_D^\intercal E_D - c_G^\intercal P_G \tag{1a}$$

$$s.t. \sum_{d \in \mathcal{D}_i} P_{D,d} + \sum_{j:(i,j) \in \mathcal{L}} \beta_{ij}(\theta_i - \theta_j) = \sum_{g \in \mathcal{G}_i} P_{G,g}, \quad \forall i \in \mathcal{N}, \tag{1b}$$

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$$\pi_{g,d} \ge 0, \ \forall g \in \mathcal{G}, \ \forall d \in \mathcal{D}. \tag{1k}$$

- When $c_D = 0$, it's equal to **standard market clearing** (carbon-agnostic).
- When c_D are identical, it's equal to adding a **unifying carbon tax on generators**.

How Non-Uniform Carbon Costs Impact?

Carbon-aware Objective

$$\max_{P_G, P_D, \theta, \pi, E_D} \quad u_D^\intercal P_D - c_D^\intercal E_D - c_G^\intercal P_G \tag{1a}$$

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$$\pi_{g,d} \ge 0, \ \forall g \in \mathcal{G}, \ \forall d \in \mathcal{D}. \tag{1k}$$

- What if $c_D \neq 0$ and the c_D values are not identical?
- How will prices change? Who pays for the higher generation cost?

Equivalent Equilibrium Formulation

Single Optimization Problem

$$\max_{P_G, P_D, \theta, \pi, E_D} \quad u_D^\intercal P_D - c_D^\intercal E_D - c_G^\intercal P_G \tag{1a}$$

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Generators: Profit maximization

$$\max_{P_{G,g}} \ (\lambda_{P,i:g\in\mathcal{G}_i} + \lambda_{G,g} - c_{G,g}) \cdot P_{G,g}$$

$$\text{s.t.} \ P_{G,g}^{\min} \le P_{G,g} \le P_{G,g}^{\max}.$$

Consumers: Utility maximization

$$\max_{P_{D,d}} \ (u_{D,d} - \lambda_{P,i:d\in\mathcal{D}_i} + \lambda_{D,d}) \cdot P_{D,d}$$

$$\text{s.t.} \ P_{D,d}^{\min} \le P_{D,d} \le P_{D,d}^{\max}.$$

Transmission Owner: Profit maximization

$$\max_{\theta} \ \sum_i \lambda_{P,i} \cdot \left(\sum_{j,(i,j)\in\mathcal{L}} \beta_{ij}(\theta_j - \theta_i) \right)$$

$$\text{s.t.} \ -F_{ij}^{\text{lim}} \le \beta_{ij}(\theta_i - \theta_j) \le F_{ij}^{\text{lim}}, \quad \forall (i,j) \in \mathcal{L},$$

$$\theta_{ref} = 0,$$

Equilibrium Formulation

Price Setter: Enforce the nodal power balance constraint

$$\sum_{d \in \mathcal{D}_i} P_{D,d} + \sum_{j:(i,j)\in\mathcal{L}} \beta_{ij}(\theta_i - \theta_j) = \sum_{g \in \mathcal{G}_i} P_{G,g}, \quad : \lambda_{P,i},$$

Carbon Manager: Total carbon cost minimization

$$\max_{\pi, E_D} \ -c_D^\intercal E_D$$

$$\text{s.t.} \ \sum_{d \in \mathcal{D}} \pi_{g,d} = P_{G,g}, \quad \forall g \in \mathcal{G}, \quad : \lambda_{G,g} \qquad \sum_{g \in \mathcal{G}} e_{G,g}\pi_{g,d} = E_{D,d}, \ \forall d \in \mathcal{D},$$

$$\sum_{g \in \mathcal{G}} \pi_{g,d} = P_{D,d}, \quad \forall d \in \mathcal{D}, \quad : \lambda_{D,d} \qquad \pi_{g,d} \ge 0, \ \forall g \in \mathcal{G}, \ \forall d \in \mathcal{D},$$

Carbon-Adjustments

Generators: Profit maximization

$$\max_{P_{G,g}} \ (\lambda_{P,i:g\in\mathcal{G}_i} + \lambda_{G,g} - c_{G,g}) \cdot P_{G,g}$$
$$\text{s.t. } P_{G,g}^{\min} \le P_{G,g} \le P_{G,g}^{\max}.$$

Consumers: Utility maximization

$$\max_{P_{D,d}} \ (u_{D,d} - \lambda_{P,i:d\in\mathcal{D}_i} + \lambda_{D,d}) \cdot P_{D,d}$$
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Transmission Owner: Profit maximization

$$\max_{\theta} \sum_i \lambda_{P,i} \cdot \left(\sum_{j,(i,j)\in\mathcal{L}} \beta_{ij}(\theta_j - \theta_i) \right)$$
$$\text{s.t. } -F_{ij}^{\lim} \le \beta_{ij}(\theta_i - \theta_j) \le F_{ij}^{\lim}, \quad \forall (i,j) \in \mathcal{L},$$
$$\theta_{ref} = 0,$$

Equilibrium Formulation

Price Setter: Enforce the nodal power balance constraint

$$\sum_{d\in\mathcal{D}_i} P_{D,d} + \sum_{j:(i,j)\in\mathcal{L}} \beta_{ij}(\theta_i - \theta_j) = \sum_{g\in\mathcal{G}_i} P_{G,g}, \quad : \lambda_{P,i},$$

Carbon Manager: Total carbon cost minimization

$$\max_{\pi, E_D} \ -c_D^\intercal E_D$$
$$\text{s.t. } \sum_{d\in\mathcal{D}} \pi_{g,d} = P_{G,g}, \quad \forall g \in \mathcal{G}, \quad : \lambda_{G,g} \qquad \sum_{g\in\mathcal{G}} e_{G,g}\pi_{g,d} = E_{D,d}, \ \forall d \in \mathcal{D},$$
$$\sum_{g\in\mathcal{G}} \pi_{g,d} = P_{D,d}, \quad \forall d \in \mathcal{D}, \quad : \lambda_{D,d} \qquad \pi_{g,d} \ge 0, \ \forall g \in \mathcal{G}, \ \forall d \in \mathcal{D},$$

Theorem (Ordering of Carbon-Adjustments):

Theorem IV.2 (Ordering of Carbon-Adjustments)**.** *For a set of generators $\mathcal{G}$ with increasing emission factors $e_{G,(1)} \le e_{G,(2)} \le \cdots \le e_{G,(|\mathcal{G}|)}$, the corresponding generator carbon-adjustments will be decreasing,*

$$\lambda_{G,(1)} \ge \lambda_{G,(2)} \ge \cdots \ge \lambda_{G,(|\mathcal{G}|)}.$$

For a set of consumers $\mathcal{D}$ with decreasing carbon-costs $c_{D,(1)} \ge c_{D,(2)} \ge \cdots \ge c_{D,(|\mathcal{D}|)}$, the corresponding consumer carbon-adjustments will be increasing,

$$\lambda_{D,(1)} \le \lambda_{D,(2)} \le \cdots \le \lambda_{D,(|\mathcal{D}|)}$$

- The higher-emitting generators are penalized with lower carbon-adjustments.
- The consumers submitting higher carbon costs contribute more to cover the increases in generation cost that arise from prioritizing low-carbon generation.

Takeaways

- We propose a new green electricity market clearing with consumer-based carbon costs and carbon allocation.
- The equivalent equilibrium formulation gives rise to carbon-adjusted prices. Low-carbon generators and carbon-sensitive consumers will face higher carbon-adjusted prices.

Scan to access paper preprint!

Thank you!

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Selected References

- Jiang, W., & Roald, L. (2025). Greening the Grid: Electricity Market Clearing with Consumer-Based Carbon Cost. IREP 2025.

- Jiang, W., Rangarajan, A., & Roald, L. (2025). Consumer-based Carbon Costs: Integrating Consumer Carbon Preferences in Electricity Markets. In submission.

Other related works from our group

Consider Carbon Emission in Market Clearing

- **Carbon tax model**

Without Carbon Tax:

$$\max_{P_G,P_D} u_D^\intercal P_D - c_G^\intercal P_G$$

With Carbon Tax:

$$\max_{P_G,P_D} u_D^\intercal P_D - (c_G^\intercal + \boldsymbol{c_{co_2} e_G^\intercal}) P_G$$

– **Change generator merit order** and make renewables competitive.

– “Carbon bill” allocation on consumers through **higher prices**.

- Politically difficult to determine an **adequate carbon tax**.
- It’s not clear to consumers that this model is the “best” choice to pay for carbon bills.

Analysis for IEEE RTS-GMLC Case

IEEE RTS-GMLC System: 73buses, 158 generators, and 120 Lines

Four generator fuel types: Natural gas(0.6), Oil (0.74), Coal (0.96), Renewable (0)

TABLE III

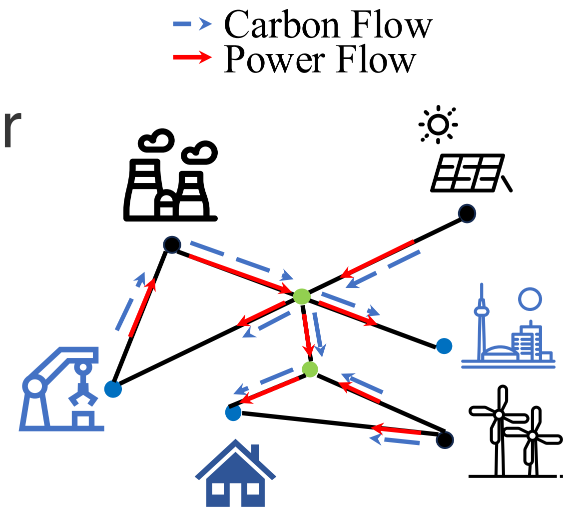
THE IMPACT OF DIFFERENT CARBON COSTS ON GENERATION DISPATCH, SYSTEM EMISSIONS, AND OBJECTIVE FUNCTION COMPONENTS.

Cases	Total Generation [MWh]	Total Generation Cost [$]	Total Carbon [tons]	Average Carbon [tons/MWh]	$u_d^\intercal P_d$ [$]	$c_{co_2}^\intercal E_d$ [$]	Optimal Objective Values [$]
P(2)	8550	63748	3001.8	0.351	457039	0	393291
P(3)	8550	63748	3001.8	0.351	457039	0	393291
$[10, 20]$	8550	64723.4	2866.9	0.335	457039	35591.4	356724.2
$[10, 40]$	8550	65468.5	2804	0.328	457039	50088.7	341481.8
$[30, 60]$	8263.3	61675.1	2594.6	0.314	448896.9	99168.8	288053
$[50, 80]$	8063.2	58907	2473.7	0.307	439138.2	143270.5	236960.7

Benchmarks: Fixed maximum loads and carbon-agnostic case when all $c_D = 0$.

Consider Carbon Emission in Market Clearing

- **Carbon flow model**
  - Carbon emissions are virtually attached to the power flowing from generators to loads.
  - Proportional sharing assumption.
  - Node carbon intensity can be calculated and limited.



Carbon Flow Model

- Difficult to determine **adequate carbon limits**.
- It’s not clear that the proportional sharing assumption is a “right” definition for carbon flow tracing.

More Extension

- **Equilibrium framework** instead of sequential framework on **carbon signal design for load shifting to reduce carbon emissions** (submitted to CDC 2025, preprint: https://arxiv.org/pdf/2504.07248).

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- **Equilibrium framework** instead of sequential framework on **carbon signal design for load shifting to reduce carbon emissions** (submitted to CDC 2025, preprint: https://arxiv.org/pdf/2504.07248).
  - **Sequential framework**[1]

Step 1: **System operator** solves DC OPF to clear the market.

Step 2: Compute locational marginal carbon emissions per bus using LP sensitivity analysis

Step 3: **Carbon-sensitive loads** determine optimal load shifts based on marginal emissions.

1. Lindberg, J., Abdennadher, Y., Chen, J., Lesieutre, B. C., & Roald, L. (2021, June). A guide to reducing carbon emissions through data center geographical load shifting. In *Proceedings of the Twelfth ACM International Conference on Future Energy Systems* (pp. 430-436).

More Extension

- **Equilibrium framework** instead of sequential framework on **carbon signal design for load shifting to reduce carbon emissions** (submitted to CDC 2025, preprint: https://arxiv.org/pdf/2504.07248).
  - **Equilibrium framework**

Generators:

$$\max_{P_{G,g}} \left(p_{i:g\in\mathcal{G}_i} - c_{G,g}\right) \cdot P_{G,g}$$

$$s.t.\ P_{G,g}^{\min} \le P_{G,g} \le P_{G,g}^{\max},$$

Consumers:

$$\max_{P_{D,d}} \left(r_{D,d} - p_{i:d\in\mathcal{D}_i} - \lambda \cdot c_{D,d}\right) \cdot P_{D,d}$$

$$s.t.\ P_{D,d}^{\min} \le P_{D,d} \le P_{D,d}^{\max},$$

Transmission Owner:

$$\max_{\theta_i} \sum_i p_i \cdot \left(\sum_{j,(i,j)\in\mathcal{L}} \beta_{ij}(\theta_j - \theta_i) \right) \quad s.t. \quad -F_{ij}^{\lim} \le \beta_{ij}(\theta_i - \theta_j) \le F_{ij}^{\lim}, \quad \forall (i,j) \in \mathcal{L}, \quad \theta_{ref} = 0,$$

Price-setter:

$$\sum_{d\in\mathcal{D}_i} P_{D,d} + \sum_{j:(i,j)\in\mathcal{L}} \beta_{ij}(\theta_i - \theta_j) = \sum_{g\in\mathcal{G}_i} P_{G,g} \quad : p$$

Average carbon emission:

$$\lambda \sum_{d\in\mathcal{D}} P_{D,d} = \sum_{g\in\mathcal{G}} e_{G,g} \cdot P_{G,g},$$

Equilibrium Analysis Introduction

- Primal Centralized Problem

$$\max_{P_g,P_d} (u^\intercal P_d - c^\intercal P_g)$$
$$s.t. \sum_{l\in\mathcal{G}} P_{g,l} \geq \sum_{l\in\mathcal{D}} P_{d,l}, \ (: \omega),$$
$$P_{g,i}^{\min} \leq P_{g,i} \leq P_{g,i}^{\max}, \ \forall i \in \mathcal{G}, \ (: v_i^{\max}, v_i^{\min}),$$
$$P_{d,i}^{\min} \leq P_{d,i} \leq P_{d,i}^{\max}, \ \forall i \in \mathcal{D}, \ (: \gamma_i^{\max}, \gamma_i^{\min}).$$

- KKT Conditions

$$0 \in c_i - \omega + v_i^{\max} - v_i^{\min}, \quad \forall i \in \mathcal{G},$$
$$0 \in \omega - u_i + \gamma_i^{\max} - \gamma_i^{\min}, \ \forall i \in \mathcal{D},$$
$$0 \leq \omega \perp \sum_{l\in\mathcal{G}} P_{g,l} - \sum_{l\in\mathcal{D}} P_{d,l} \geq 0,$$
$$0 \leq v_i^{\max} \perp P_{g,i}^{\max} - P_{g,i} \geq 0, \forall i \in \mathcal{G},$$
$$0 \leq v_i^{\min} \perp P_{g,i}^{-} P_{g,i}^{\min} \geq 0, \forall i \in \mathcal{G},$$
$$0 \leq \gamma_i^{\max} \perp P_{d,i}^{\max} - P_{d,i} \geq 0, \forall i \in \mathcal{D},$$
$$0 \leq \gamma_i^{\min} \perp P_{d,i} - P_{d,i}^{\min} \geq 0, \forall i \in \mathcal{D}.$$

Equivalent

- Primal Equilibrium Problem

Generators: $\max_{P_{g,l}} (p - c_l) P_{g,l}$

$$s.t. \ P_{g,l}^{\min} \leq P_{g,l} \leq P_{g,l}^{\max}, \ (: v_i^{\max}, v_i^{\min}).$$

Consumers: $\max_{P_{d,l}} (u_l - p) \cdot P_{d,l}$

$$s.t. \ P_{d,l}^{\min} \leq P_{d,l} \leq P_{d,l}^{\max}, \ (: \gamma_i^{\max}, \gamma_i^{\min}).$$

Price-setter Problem: $0 \leq p \perp \sum_{l\in\mathcal{G}} P_{g,l} - \sum_{l\in\mathcal{D}} P_{d,l} \geq 0$

- KKT Conditions

$$0 \in c_i - p + v_i^{\max} - v_i^{\min}, \quad \forall i \in \mathcal{G},$$
$$0 \in p - u_i + \gamma_i^{\max} - \gamma_i^{\min}, \ \forall i \in \mathcal{D},$$
$$0 \leq p \perp \sum_{l\in\mathcal{G}} P_{g,l} - \sum_{l\in\mathcal{D}} P_{d,l} \geq 0,$$
$$0 \leq v_i^{\max} \perp P_{g,i}^{\max} - P_{g,i} \geq 0, \forall i \in \mathcal{G},$$
$$0 \leq v_i^{\min} \perp P_{g,i}^{-} P_{g,i}^{\min} \geq 0, \forall i \in \mathcal{G},$$
$$0 \leq \gamma_i^{\max} \perp P_{d,i}^{\max} - P_{d,i} \geq 0, \forall i \in \mathcal{D},$$
$$0 \leq \gamma_i^{\min} \perp P_{d,i} - P_{d,i}^{\min} \geq 0, \forall i \in \mathcal{D}.$$