Robust Optimization with Statistically Feasible Guarantee

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Decision-Making under Uncertainty

Deterministic case:

$$\begin{aligned} &\min\ f(x) \\ s.t.\ &g(x;\xi) \le 0 \end{aligned} \qquad (1)$$

Decision vector $x \in \mathbb{R}^d$.
fixed parameter vector $\xi \in \mathbb{R}^m$.

Decision-making under uncertainty:

$\xi \in \mathbb{R}^m$: from fixed parameters to random vectors.

Uncertainties in power system operation: e.g., renewable power generation, electricity consumption, prices, etc.

Conventional Wisdom

$$\min \; f(x) \quad s.t. \; g(x;\xi) \le 0 \qquad (2)$$

Decision vector $x \in \mathbb{R}^d$.
Random vector $\xi \in \mathbb{R}^m$.

- If ξ satisfies some specific distribution, e.g., Gaussian distribution, we can uses chance-constrained programming (CCP)

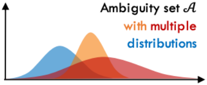
$$\min f(x), \quad s.t. \; \mathbb{P}_{\xi\sim P}(g(x;\xi) \le 0) \ge 1-\epsilon. \qquad (3)$$

Often require distribution information (either assumed or estimated)

- Distributionally Robust Optimization (DRO)

$$\min f(x), \quad s.t. \; \min_{P\in\mathcal{A}} \mathbb{P}_{\xi\sim P}(g(x;\xi) \le 0) \ge 1-\epsilon. \qquad (4)$$

Suffer from the ambiguity set construction and computation efficiency issues



Conventional Wisdom

- Scenario Generation

$$\min f(x), \quad s.t.\ g(x;\xi_i) \leq 0, \xi_1, \cdots, \xi_N. \tag{5}$$

Enumeration, time/cost-consuming

- Robust Optimization (RO)

$$\min f(x), \quad s.t.\ g(x;\xi) \leq 0, \xi \in \mathcal{U}(D). \tag{6}$$

Suffer from conservation issue

- ...

Utilize RO to Approximate CCP Solutions

CCP:

$$\min \; f(x)$$
$$s.t. \; P_\xi(g(x;\xi) \le 0) \ge 1-\epsilon.$$

RO:

$$\min \; f(x)$$
$$s.t. \; g(x;\xi) \le 0, \; \xi \in \mathcal{U}.$$

Feasibility Guarantee: Given a dataset Ψ, $\mathcal{U}$ is an uncertainty set covering $\ge 1-\epsilon$ samples of Ψ.

- For any feasible solution of RO, i.e., $\hat{x}$, we have $P_\xi(g(\hat{x};\xi) \le 0) \ge P_\xi(\xi \in \mathcal{U})$.
- $\hat{x}$ can only guarantee the chance constraint violation for this given dataset. (No generalization guarantee)

Statistical Feasibility Guarantee [1]: If $\mathcal{U}$ can guarantee that for any new dataset $\tilde{\Psi}$, the solution $\hat{x}$ from RO makes $P_{\tilde{\Psi}}(P_\xi(g(\hat{x};\xi) \le 0) \ge 1-\epsilon) \ge 1-\beta$ hold. (Generalization guarantee)

[1] Hong, L.J., Huang, Z., and Lam, H. "Learning-based robust optimization: Procedures and statistical guarantees" . In: Management Science 67.6 (2021), pp. 3447–3467.

Robust Uncertainty Set Construction

Goal: Obtain an uncertainty set $\mathcal{U}$ such that $P_\Psi(P_\xi(\xi \in \mathcal{U}(\Psi)) \geq 1-\epsilon) \geq 1-\beta$.

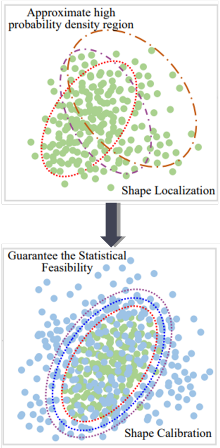
Two-stage strategy: Given dataset $\Psi = \{\xi_1, \xi_2, \cdots, \xi_n\}$, separated into Ψ_1 and Ψ_2 (with size m_1 and m_2, $m_1 + m_2 = n$).

Stage 1 (Shape Localization):

- Use Ψ_1 to learn the shape of the uncertainty set, obtain $\mathcal{U}_0$ (for tractability, the shape can be ellipsoids or polytypes).

Stage 2 (Shape Calibration):

- Reformulate the uncertainty set as $\mathcal{U}_0 = \{\xi : h(\xi) \leq s_0\}$, where s_0 is a scalar.
- Use Ψ_2 to resize the set by calibrating a new value s in $\mathcal{U} = \{\xi : h(\xi) \leq s\}$.



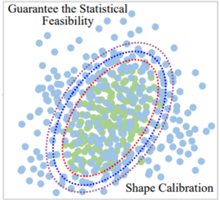
Size Calibration for Statistical Feasibility Guarantee

Shape Calibration:

- Given dataset $\Psi_2 = \{\xi_1, \xi_2, \cdots, \xi_{m_2}\}$, and an uncertainty set $\mathcal{U}_0 = \{\xi : h(\xi) \le s_0\}$.
- Order the observations $h(\xi_{(1)}) < h(\xi_{(2)}) < \cdots < h(\xi_{(m_2)})$.
- Find i^* such that

$$i^* = \min\left\{ r : \sum_{k=0}^{r-1} \binom{m_2}{k} (1-\epsilon)^k \epsilon^{m_2-k} \ge 1-\beta\ , \quad 1 \le r \le m_2 \right\}.$$

- Set $s = h(\xi_{(i^*)})$, and $\mathcal{U} = \{\xi : h(\xi) \le s\}$.



Theorem 1: $\mathcal{U} = \{\xi : h(\xi) \le s\}$ satisfies $P_\Psi(P_\xi(\xi \in \mathcal{U}(\Psi)) \ge 1-\epsilon) \ge 1-\beta$, if $m_2 \ge \frac{\log \beta}{\log(1-\epsilon)}$.

Self-Improving Uncertainty Set Reconstruction

CCP:

$$\min \; f(x)$$
$$s.t. \; P_\xi(g(x;\xi) \leq 0) \geq 1 - \epsilon.$$

Assumption: Suppose the optimal solution to CCP is x^*, then

$$P_\xi(g(x^*;\xi) \leq 0) \geq 1 - \epsilon.$$

A New Uncertainty Set:

$\mathcal{U}^* = \{\xi : g(x^*;\xi) \leq 0\}$. Clearly,

$$P_\xi(g(x^*;\xi) \leq 0) = P_\xi(\xi \in \mathcal{U}^*) \geq 1 - \epsilon.$$

Self-Improving Uncertainty Set Reconstruction Procedure:

- Use Ψ_1 to obtain the approximated optimal solutions x_0 with the previous two-stage strategy. $\mathcal{U}^* = \{\xi : g(x_0;\xi) \leq 0\}$.
- Calibrate $\mathcal{U}^*$ by following **Shape Calibration** (stage 2 in the Robust Uncertainty Set Construction) with Ψ_2.

Some Application Settings

Starting from: Providing a statistically feasible solution for RO.

Aimed at no prior knowledge on randomness, we explore

- EV charging scheduling[1].
- TCLs scheduling[2].

This framework can also be extended to deal with:

- Joint chance constraints[3].
- Economic dispatch[3] and Unit commitment[4].
- ...

[1] Jiang, W., Liang, J., Lu, C., and Wu, C. "Robust Online EV Charging Scheduling with Statistical Feasibility" . In: 2023 IEEE 55th Conference on Decision and Control (CDC). IEEE.

[2] Jiang, W., Lu, C., and Wu, C. "Robust Scheduling of Thermostatically Controlled Loads with Statistically Feasible Guarantees" . In: IEEE Transactions on Smart Grid (2023).

[3] Lu, C., Gu, N., Jiang, W., and Wu, C. "Sample-Adaptive Robust Economic Dispatch With Statistically Feasible Guarantees" . In: IEEE Transactions on Power Systems (2023).

[4] Liang, J., Jiang, W., Lu, C., and Wu, C. "Joint chance-constrained unit commitment: Statistically feasible robust optimization with learning-to-optimize acceleration" . In: IEEE Transactions on Power Systems (2024).

How about the Performance?

Table: Single-period results for different methods with $\delta = 0.05,\ \xi = 0.05$.

	$REP(‰)$	$\tilde{\delta}$	$\tilde{\xi}$	$CPU\ time(s)$
RSO	7.676	0	0	0.262
RRSO	**4.073**	**0.068**	**0.071**	0.489
SG	9.237	0	0	0.229
CC	7.825	0	0	0.257
DM	-0.863	0.078	0.193	0.204

- RRSO achieves the lowest REP, and controls the violation rate close to the target.

Conclusions and Take-Away

- We introduce two uncertainty set construction methods to guarantee statistical feasible of RO-based solutions (balance the trade-off between solution optimality and robustness).
- One method decouples the uncertainty set construction from the downstream optimization, while the other couples them by explicitly incorporating the downstream problem's structure.

Thank you for your listening!

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Backup Slide 1: About Evaluation Matrics

Over 200 testing cases, we compare

- TCC: the total charging cost.
- ACP: the average charging unit price.
- ASP: the average shortening period (the mean value of the shortening time comparing the EVs' departure time).
- SDR: the service drop rate, which is calculated as follows:

$$\mathrm{SDR} = 1 - \frac{\text{the number of feasible solutions}}{200}, \tag{7}$$

where the feasible solutions are required to satisfy Eq. (9) for $\forall t \in [2, T]$.

For example, if CC obtains 180 solvable cases and only 104 feasible cases which succeed in handling the future EVs' demands, then TCC, ACP, and ASP is calculated by 180 solvable cases while $\mathrm{SDR} = 1 - 104/200$.

Statistical Feasibility

Definition (Statistical Feasibility[1])

Consider a CCP,

$$\min f(x), \quad s.t.\ P_\xi(g(x;\xi) \leq 0) \geq 1-\epsilon. \tag{8}$$

Assume a dataset $\Psi = \{\xi_1, \xi_2, \cdots, \xi_n\}$ is sampled from a probability measure $\mathcal{P}$ and $\chi(\Psi)$ is constructed based on Ψ. An algorithm is statistically feasible if the resulting solution x^* is feasible for the chance constraint with confidence $1-\beta$, i.e.,

$$P_\Psi(P_\xi(g(x^*;\xi \in \chi(\Psi)) \leq 0) \geq 1-\epsilon) \geq 1-\beta. \tag{9}$$

- Given a dataset, an algorithm provides a solution x^*. (We can consider x^* as a mapping of data)
- Statistical feasibility implies that the optimal solutions guarantee at least $1-\beta$ confidence level for the chance constraints given any dataset.

[1] Hong, L.J., Huang, Z., and Lam, H. "Learning-based robust optimization: Procedures and statistical guarantees". In: Management Science 67.6 (2021), pp. 3447–3467.

Backup Slide 2: Differences between Our Method and Conventional methods

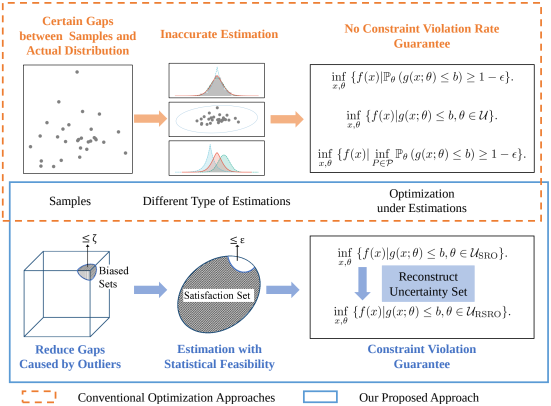

Figure: Our Approach v.s. Its Conventional Rivals.

Statistically Feasible Guarantee

$$\mathbb{P}_{\Psi_2}(P(\xi \in \mathcal{U}(\Psi)) \geq 1-\epsilon \big| \Psi_1) = \mathbb{P}_{\Psi_2}(P(h(\xi) \leq h(\xi_{i^*})) \geq 1-\epsilon \big| \Psi_1)$$
$$= \mathbb{P}_{\Psi_2}(P(q_{1-\epsilon} \leq h(\xi_{i^*})) \geq 1-\epsilon \big| \Psi_1) \geq 1-\beta \quad (10)$$

as long as $m_2 \geq \frac{log\beta}{\log(1-\epsilon)}$

Note that (10) holds given any realization of Ψ_1. Thus, taking expectation with respect to Ψ_1 on both sides in (10), we have

$$\mathbb{E}_{\Psi_1}(\mathbb{P}_{\Psi_2}(P(q_{1-\epsilon} \leq h(\xi_{i^*})) \geq 1-\epsilon \big| \Psi_1)) \geq 1-\beta \quad (11)$$

Using Law of Total Probability:

$$\mathbb{P}_{\Psi}(P(\xi \in \mathcal{U}(\Psi) \geq 1-\epsilon) \geq 1-\beta \quad (12)$$

Example (Ellipsoid Uncertainty Set)

RO:

$$\min\ f(x)$$
$$s.t.\ \xi' x \leq b,\ \xi \in \mathcal{U} = \{\xi : (\xi - \mu)\Sigma^{-1}(\xi - \mu) \leq s\}$$

↓

Deterministic Equivalent:

$$\min\ f(x)$$
$$s.t.\ \mu' x + \sqrt{s}||\Sigma^{1/2} x||_2 \leq b.$$

A Simple Application [1]: TCLs scheduling

Stochastic Factors

$$\min_{\mathbf{u}} \sum_{t=t_0}^{t_0+N-1} p_t \cdot |u_t|$$

$$s.t.\ T_{t+1} = \alpha T_t + (1-\alpha)(T_t^a - Ru_t),\ \forall t$$

$$-U_{tr} \leq u_t \leq U_{tr}, \forall t$$

$$T_{set} - \kappa \leq T_{t+1} \leq T_{set} + \kappa, \forall t$$

Cost-effective Decision $\mathbf{u}$

The Room Temperature State Evolution

the TCL' s Energy Transfer Rate Limit

The Primary Source of Flexibility for TCLs scheduling

Control action turns into
$\tilde{u}_t = u_t + a_t \omega_t$

T_t: the room temperature at time t;
u_t: the control action of TCL at time t;
T_t^a: the outdoor temperature at time t;
p_t: the electricity price at time t;
$\alpha, R, U_{tr}, T_{set}, \kappa$: deterministic parameters for a TCL;

$\omega_t = T_t^{ar} - T_t^a$: the outdoor temperature prediction error;